VELOCITY AND ACCELERATION CALCULATION FOR 3D TRAJECTORIES IN INSIGHT V3V™ SOFTWARE

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TECHNICAL NOTE (A4)

Definition of a 3D Trajectory

A three-dimensional trajectory is a linked set of particles in 3D that appear in N_f consecutive time frames. Each particle's position is determined via the triangulation of particle images found in N_i number of images. Trajectories are formed when Particle Identification and Reconstruction (DPIR) has access to time-resolved data. Since "time-resolved" refers to the ability to resolve the fastest turbulence of interest, INSIGHT V3V software assumes that trajectories are smooth and thus fits a polynomial to each 3D trajectory. Using this polynomial, INSIGHT V3V software can recalculate a particle's position, correcting for error due to particle identification and triangulation. Recalculated positions are then used for determining velocity and acceleration along a 3D trajectory.

Velocity and Acceleration

INSIGHT V3V™ Volumetric 3-Component Flow Measurement and Visualization Software utilizes 2nd order accurate (meaning that the truncation error is at least of order Δt^2) finite differencing whenever possible among trajectory members to calculate the derived vectors of velocity and acceleration.

Velocity, U_i is a vector that can be calculated two ways. When using time-resolved data, INSIGHT V3V software uses 3D trajectories. When using dual-frame data, INSIGHT V3V software uses traditional particle tracking schemes (see [2] and [3]). Here we present the method used with 3D trajectories. All velocities are 2nd order accurate.

a. Backwards Differencing

$$U_i 2\Delta t = \frac{\partial x_i}{\partial t} 2\Delta t = x_i^{t-2} - 4x_i^{t-1} + 3x_i^t$$

b. Central Differencing

$$U_i 2\Delta t = \frac{\partial x_i}{\partial t} 2\Delta t = x_i^{t+1} - x_i^{t-1}$$

c. Forwards Differencing

$$U_i 2\Delta t = \frac{\partial x_i}{\partial t} 2\Delta t = -x_i^{t+2} + 4x_i^{t+1} - 3x_i^t$$

Acceleration, A_i is a vector that can only be calculated when 3D trajectories are available. When $N_f = 3$, acceleration at the ends of the trajectory are 1st order accurate. Otherwise, accelerations are 2nd order accurate.

a. Backwards Differencing, Order (Δt)

$$A_i \Delta t^2 = \frac{\partial^2 x_i}{\partial t^2} \Delta t^2 = x_i^{t-2} - 2x_i^{t-1} + x_i^t$$

b. Backwards Differencing, Order (Δt^2)

$$A_i \Delta t^2 = \frac{\partial^2 x_i}{\partial t^2} \Delta t^2 = -x_i^{t-3} + 4x_i^{t-2} - 5x_i^{t-1} + 2x_i^t$$

c. Central Differencing, Order (Δt^2)

$$A_i \Delta t^2 = \frac{\partial^2 x_i}{\partial t^2} \Delta t^2 = x_i^{t-1} - 2x_i^t + x_i^{t+1}$$

d. Forwards Differencing, Order (Δt)

$$A_i \Delta t^2 = \frac{\partial^2 x_i}{\partial t^2} \Delta t^2 = x_i^t - 2x_i^{t+1} + x_i^{t+2}$$

e. Forwards Differencing, Order (Δt^2)

$$A_i \Delta t^2 = \frac{\partial^2 x_i}{\partial t^2} \Delta t^2 = 2x_i^t - 5x_i^{t+1} + 4x_i^{t+2} - x_i^{t+3}$$

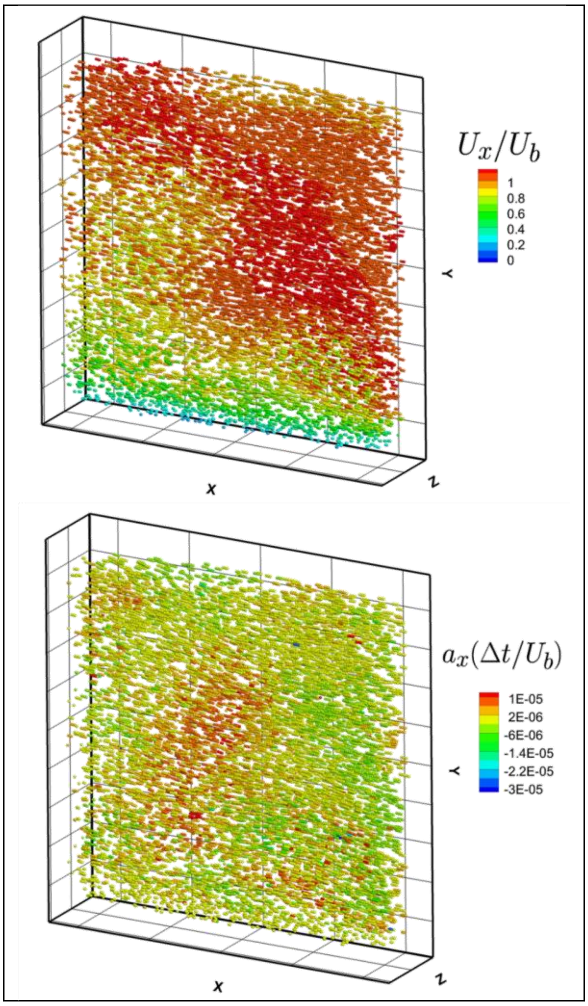

Figure 1. Turbulent boundary layer with particles in 3D trajectories, colored by: (Top) Streamwise component of normalized velocity. (Bottom) Streamwise component of normalized acceleration.

References

[1]Boomsma, A., & Troolin, D., 2018, "Time-resolved particle image identification and reconstruction for volumetric 4D-PTV," 19th Lisbon Intl. Laser Symposium.

[2]Ohmi K., & Li, H.Y., 2000, "Particle-tracking velocimetry with new algorithms," MST 11.6.

[3]Stellmacher M., & Obermayer K., 2000, "A new particle tracking algorithm based on deterministic annealing and alternative distance measures," *Exp. In Fluids*, **28**: 506-507.

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