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V3V Volumetric PIV: New Developments in Particle Reconstruction

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ABSTRACT

Every approach to volumetric PIV must address the issue of particle, or object, reconstruction. In the current study, we discuss common sources of reconstruction error in detail and describe three enhancement techniques to traditional reconstruction methods. Using a Volumetric 3-component Velocimetry (V3V) instrument, we first validate the Weak Particle Reconstruction method, which reduces lost particles due to non-uniform lighting. Second, we reclaim, on average, 5% more particles due to particle image overlap using Neighbor Tracking Reconstruction. And third, we use experimental data to show that the Auto-Calibration Reconstruction method drives down calibration errors from 0.5 pixels to about 0.1 pixels.

1. Introduction & Background

Volumetric Particle Image Velocimetry (PIV) has become an established technique to obtain velocity vectors from a three-dimensional volume. In general, there are three approaches to volumetric three-component velocity measurements (listed chronologically): 1) holographic PIV, 2) volumetric Particle Tracking Velocimetry (PTV), and 3) tomographic PIV. In holographic PIV (Barnhart et al. 1994), one attempts to reconstruct particle images into three-dimensional space by recording interference patterns created by scattered light from particles in the flow and a reference light source. This technique has very high spatial resolution, but a limited field of view. Velocity vectors are obtained via particle tracking algorithms. Typical measurement depth is several mm. Volumetric PTV was originally described by Willert & Gharib (1992) and refined by Pereira et al. (2000). It is often carried out with three cameras, though more or less can be used. The method utilizes triangulation to reconstruct particle images to a point in three-dimensional space, where one particle from each aperture is required for reconstruction. Velocity vectors are obtained via particle tracking algorithms. Typical measurement depth is much larger than other techniques, at approximately 100 mm. Elsinga et al. (2006) first detailed tomographic PIV. Commonly performed with four cameras (though more or less can be used), this technique also attempts to project particle images into a three-dimensional space. The results of such projections (i.e., object reconstruction) are objects in space that are subsequently correlated in three dimensions. Typical out-of-plane measurement depth is approximately 10 mm. A common thread among each methodology is some type of three-dimensional reconstruction of particle images, and that is the subject of this paper.

In the present study, we utilize a Volumetric 3-component Velocimetry (V3V) system—a volumetric PTV technique. With V3V, three cameras are mounted in a triangular support on a single plane to track tens of thousands of particles in a volume up to 14x14x10cm^3. Figure 1 shows the field of view common to all three apertures by overlaying an image frame from each camera. Due to the configuration of the cameras and each aperture's own line-of-sight, a particle will appear spatially shifted in each image. When all three frames are overlaid, the particle will be seen as a triangle with some characteristic length. This is a useful attribute of photogrammetry because the side-length of the triangle is a function of the particle's depth (i.e., the distance from the camera plane). As the particle nears the camera plane, the triangle's length increases, and vice versa.

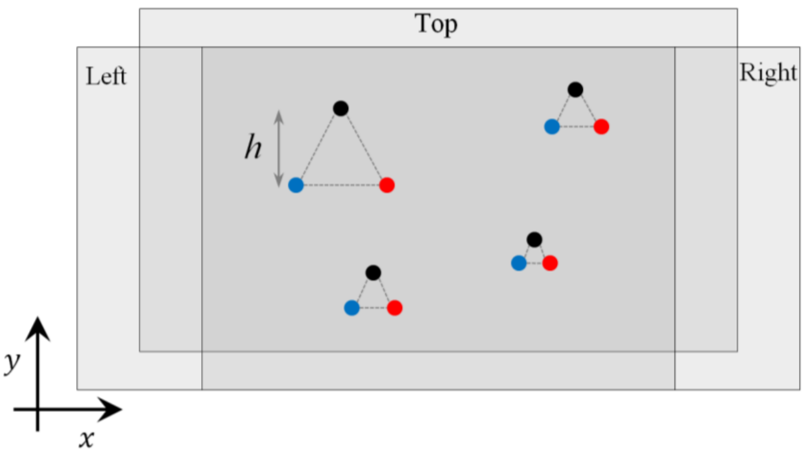

Figure 1. Overlaid top, left, and right image frames. Particles from top (black), left (blue), and right (red) frames. Apparent triangles with characteristic height, *h*.

Because V3V is a particle tracking technique, it is of great importance to correctly identify the same particle in each camera frame i.e., each particle image must be accurately reconstructed in three-dimensional space prior to particle tracking. While accuracy is always prioritized, reconstruction algorithms should also be robust enough to reconstruct as many particles as possible in order to provide high resolution experimental data. Therefore, in the present study, we present three techniques used to enhance an already existing volumetric PTV particle reconstruction method in an effort to increase the number of usable particles for tracking methods.

2. Investigated Methods of 3D Particle Reconstruction

We herein describe and implement three algorithms to compliment an existing particle triangulation method for volumetric PTV. First, we describe the existing reconstruction method, define the challenges associated with reconstruction, and then proceed to detail each enhancement method.

2.1 Volumetric PTV Particle Reconstruction

Our existing particle reconstruction is often termed the Triplet Search, since the experimental setup utilizes three cameras, though as mentioned, more or less apertures can be used during particle reconstruction. The fundamental goal of the triplet search is to overlap, in three-dimensional space, a particle image from each camera. One crucial component for a successful triplet search is an accurate calibration, which acts as a mapping function from pixel space to physical space (the calibration can also be used to map in reverse; from physical to pixel space). The calibration consists of a calibration target translating in a plane normal to the cameras. The target consists of a grid of calibration points, whose three-dimensional position is well known. After the calibration points have been identified as particle images, the calibration mapping is created by using a least squares fit to map each particle image to real-world space. Using V3V, the calibration error of the least squares fit is typically less than 0.25 pixels. The calibration also serves the purpose of correcting optical distortions. Note that the triplets (i.e., a three-dimensional reconstructed particle from three particle images) shown in Figure 1 are equilateral, which is due to aberration correction by the calibration.

With a successful mapping function acquired, the triplet search proceeds methodically by mapping every particle image from each aperture to physical space. This mapping occurs on the calibration planes (and intermediary planes). A triplet is created when particle images from each aperture overlap in physical space on some depth-wise plane. Since the calibration is a least squares fit, and because the cameras can inadvertently be moved spatially (temperature, vibration, etc…) between the time of calibration and measurement, particle image overlap is allowed to occur within a specified tolerance. The three-dimensional position of the triplet in *x* and *y* directions (refer to Figure 1) is determined by the mapping to physical space, but the out-of-plane position, *z*, is determined by the characteristic height of the triangular triplet and the calibration. Because any given particle within the flow should only be perceived once in each aperture, when a triplet is reconstructed using a given particle image, that image should not be used as a member of any other triplet.

2.2 Sources of Particle Reconstruction Failure

Particle reconstruction failure is often caused by a three common sources: 1) calibration error; 2) lighting non-uniformities; 3) poor particle identification. Calibration error is well known and can be a consequence of the algorithm, but it is most commonly caused by an inadvertent movement of the camera/experimental apparatus. Non-uniform lighting can be due to several experimental considerations including a varying intensity distribution of the light source, shadows cast within the flow, reflections, and high background intensity, among others. Significant lighting variations can cause reconstruction failures because during particle identification, an intensity threshold is commonly utilized to delineate between particle and noise. As such, some particles within the flow volume are well lit, and for others that are not, particle reconstruction may fail. One specific non-uniformity is a difference in light intensity between frames. Occurring randomly throughout time, one frame can have a significantly higher average intensity than another leading to poor particle identification in the dimmer frame.

Poor particle identification has a number of experimental causes including pixel intensity thresholding, particle conglomeration, and the overlapping of particles. Pixel intensity thresholding (aka particle washout) effectively reduces the accuracy of particle identification because the brightest part of a particle image is thesholded. As a result, information about the particle is lost and its center and radius can be inaccurate. Particle conglomeration, or the grouping of particles, leads to particle images that are not Gaussian. Finally, since particle reconstruction relies on the assumption that a particle in the flow should be seen once by each aperture, if in one aperture, a particle image overlaps another, a triplet cannot be created. Overlapping particles are a consequence of aperture line-of-sight and will occur randomly as it is a function of local particle velocity and position.

With these potential failure modes in mind, we now present three enhancement methods to complement traditional particle reconstruction: 1) weak particle reconstruction, 2) neighbor tracking reconstruction, and 3) auto calibration reconstruction.

2.2 Weak Particle Reconstruction (WPR)

WPR is a tool used to address lighting non-uniformities. In any photogrammetric set of images, particles will have various peak intensities, some below and some above a prescribed threshold. Particles with peak intensities below the threshold are often termed *weak particles*, whereas those above the threshold are termed *strong particles*. Usually, weak particles are not used during particle reconstruction due to the chance that they could be associated with noise in the image, rather than actual particles. As originally described by Ponchaut (2005), WPR occurs when two strong particles overlay in the physical plane, but a third (or however many cameras are used) cannot be found. In this situation, a pool of weak particles will be sampled to locate a match. The primary advantage of WPR is that the minimum intensity threshold for the entire flow field need not be reduced to accompany low lit particles in some regions of the flow. Weak particles are still identified particles, and therefore, also have some intensity threshold so as not be confused with background noise. The level of this threshold is dependent upon the experiment (source lighting, background lighting, shadows, etc.).

2.3 Neighbor Tracking Reconstruction (NTR)

NTR is a tool used to address particle overlap and poor particle tracking. In the current work, six total images were taken with our photogrammetric system—three at some time, t and another three at time $t + \Delta t$. Here, Δt is the laser pulse delay time and is used to calculate the particle velocity, as the displacement is known after particle reconstruction and tracking. Consider particles in the flow, A and B, as shown in Figure 2. On the left is shown case in which both particles at time t (grey spheres) are matched with corresponding particles at time $t + \Delta t$ (orange spheres). Subsequently, velocity vectors can be calculated for each. Now consider the case shown on the right. A match has been found for particle B, but not for A. NTR seeks to locate a triplet for particle A at time $t + \Delta t$ by performing the triplet search again, except only at a probable location, which is termed a search location, or $\overrightarrow{L_s}$, a three-dimensional positional vector. $\overrightarrow{L_s}$ is represented by the red dot in Figure 2. The accompanying dashed box represents searching bounds around the search location, which are editable, but in present study, have been chosen as 8 pixels wide (and tall).

The probable location of the missing triplet at time $t + \Delta t$ can be approximated by calculating a local velocity vector and multiplying by Δt to obtain a displacement. Therefore, obtaining an approximated local vector is required. We initially sought to perform a two-dimensional cross-correlation using particle images that belong to triplets near particle A (see Figure 2), but determined that the use of an Inverse Distance Weighting (IDW) function, specifically that of Shepard (1968) to be far more robust. The IDW function is a type of point cloud interpolation method, which is a common research problem, related to meshless computational fluid dynamics methods (see Belytschko et al. 1996). Referring again to Figure

2, we take the following approach: velocity vector $\overrightarrow{V_B}$ is assigned to particle B. An IDW function interpolates $\overrightarrow{V_B}$ onto particle A, producing an intermediate velocity vector, $\overrightarrow{V_A'}$ and the search location is calculated as:

$$\overrightarrow{L_s} = \Delta t\, \overrightarrow{V_A'}$$

To account for potential particle overlap and poor particle tracking, the traditional triplet search is performed again, though only within the bounds of $\overrightarrow{L_s}$, as shown in Figure 2. Of course, this explanation has been simplified for illustrative purposes. In application, $\overrightarrow{V_A'}$ would be calculated from all surrounding neighbors of particle A. In this way, trajectories from neighboring particles are used to reconstruct a match for particle A at time $t + \Delta t$.

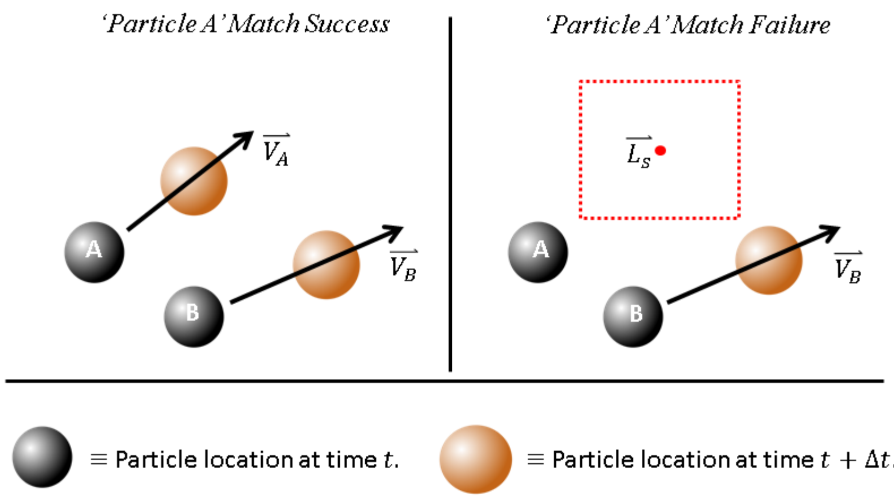

Figure 2. An illustration of Neighbor Tracking Reconstruction method.

2.4 Auto-Calibration Reconstruction (ACR)

The ACR method addresses reconstruction failure due to calibration error by comparing the projections of reconstructed triplets (onto image planes) and the original particle images of those triplets. In the past, other methods, such as stereo calibration (Bjorquist, 2000) and self-calibration (Weineke, 2008) have also sought to reduce calibration error by comparing particle images with and without the use of a given calibration.

The goal of the ACR method is to modify an existing calibration (or mapping function) to account for errors therein. As stated in Section 2.2, while calibration errors can be due to algorithm error, it is often the case that cameras get moved between calibrations. Errors due to calibration can be calculated by projecting already reconstructed triplets in three-dimensional space to each aperture's image plane. The resulting pixel location on the image plane is subtracted from the triplet's original particle image location. This difference is termed an error vector, $\overrightarrow{e_i}$, where subscript *i* refers to the aperture number. This process is illustrated in Figure 3. In volumetric PTV, the reason that the original particle image does not overlay with the projected image is because the three-dimensional location is not solely determined by just a single particle image in a single aperture image frame. Instead, its 3D position is determined by three particle images, each of which will have a slightly different location in real-world space (recall that a triplet is created when particles overlap *within some tolerance* in three-dimensional space).

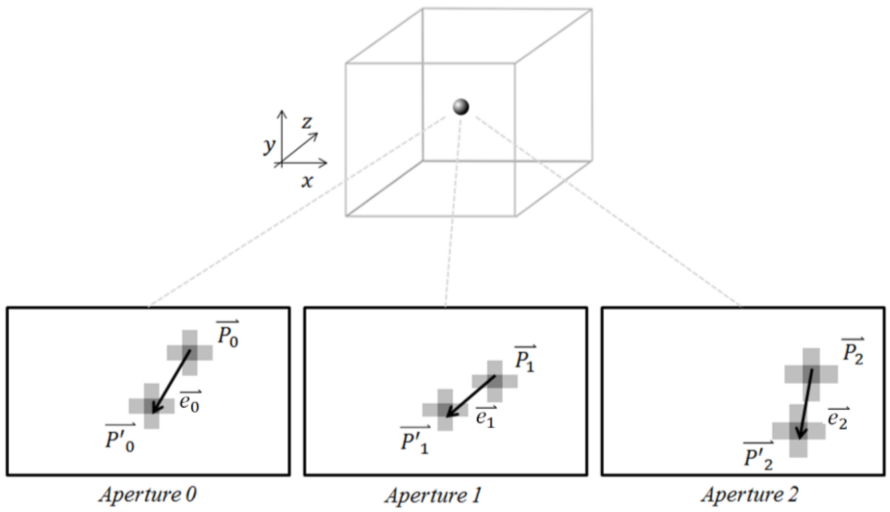

Figure 3. A particle in three-dimensional space has original particle image locations $\vec{P}_i$, though the projection of the particle on to each aperture frame is denoted by $\vec{P}'_i$.

The procedure detailed in Figure 3 is repeated for every reconstructed triplet. To correct the calibration, ACR utilizes a statistical analysis of the error vectors to calculate local average error vectors at each calibration plane, which are then used to correct the calibration. Multiple passes of the method can occur, in order to drive down error.

3. Results & Discussion

3.1 Weak Particle Reconstruction (WPR)

As a verification of the WPR method, we have taken raw experimental images from the work of Troolin & Longmire (2010), as shown in Figure 4a, and artificially reduced the image intensity by a factor of three for *a single image* (the image from the second aperture) as shown in Figure 4b (highlighted with red), so as to simulate non-uniform lighting among the three photogrammetric views.

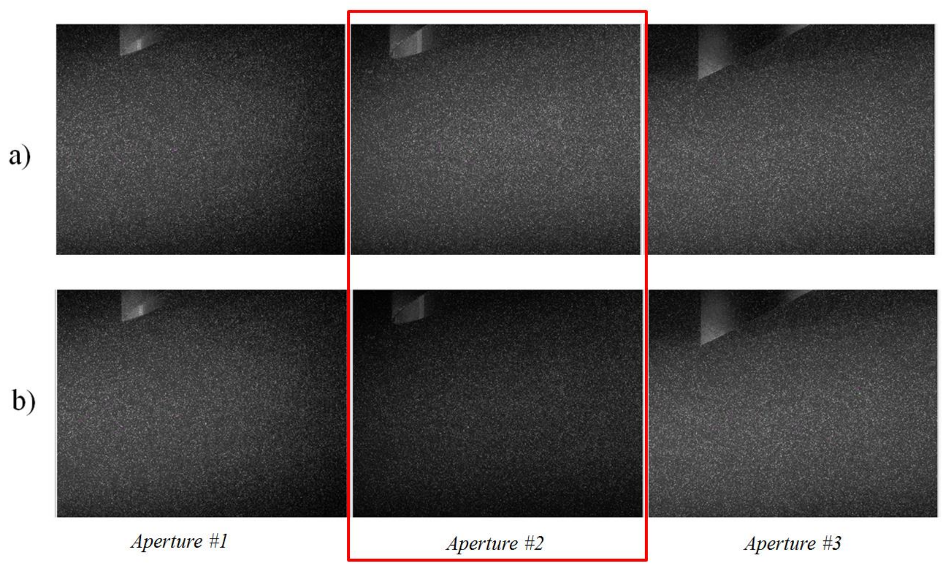

Figure 4. Experimental data from Troolin & Longmire (2010).
a) Original, raw images. b) Image from aperture #2 has been artificially dimmed by a factor of three.

Using these two image sets, we conducted triplet reconstruction three different ways:

- **Case A:** Original data set. Particle identification intensity threshold of 400. No WPR method.
- **Case B:** Aperture #2 intensity reduced by factor of three. Particle identification intensity threshold of 400. No WPR method.
- **Case C:** Aperture #2 intensity reduced by factor of three. Particle identification intensity threshold of 132 (i.e., 400/3), with WPR threshold of 132.

Figure 5 details particle image counts in each aperture's image and the corresponding number of reconstructed triplets for each case. Case A is the experimental control—the original images with a particle ID threshold of 400. Approximately 22k triplets are reconstructed. Case B represents the scenario in which non-uniform lighting has occurred, and no WPR method is utilized. As a result of the non-uniform lighting in aperture #2, only 11k triplets were found. The cause is clearly seen as due to loss of particle images in aperture #2 (as expected). Lastly, in Case C, we have included particles from each aperture with an intensity of at least 132 (i.e., 400/3). Accordingly, increased particle counts are seen for each aperture. In order to reconstruct the lost triplets of Case B, we used WPR to define weak and strong particles. Weak particles were defined by those with a peak intensity less than 400—strong particles were those with an intensity greater than 400. As a result of WPR, the lost triplets due to a non-uniform lighting were recovered (and more), as approximately 24k triplets were found.

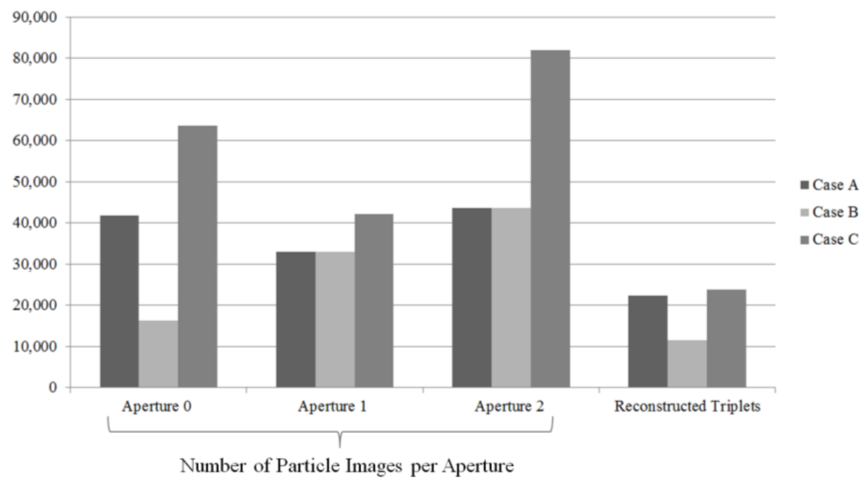

Figure 5. Particle images per aperture and total number of reconstructed triplets for each case.

3.2 Neighbor Tracking Reconstruction (NTR)

As a preliminary assessment of the NTR method, we reprocessed experimental data from six in-house cases. Of particular importance to this assessment was to quantify the increase in reconstructed triplets. As described in Section 2.3, NTR utilizes an IDW interpolation function to calculate the search locations. One of the six test cases included the experimental data of Troolin & Longmire (2010). Matched triplets (those that have a corresponding velocity due to successful particle tracking) are shown in Figure 5a as individual points. Viewing from the top, the points are colored by magnitude of velocity. Figure 5b shows triplets that were not matched during particle tracking. As such, NTR is performed for each and its search location is determined by the local velocity from triplets in Figure 5a. We have colored each unmatched triplet with its IDW interpolated local velocity. Threfore, triplets in Figure 5b should have a velocity distribution similar to that seen in Figure 5a, which as shown, is the case. Once the search locations are identified, traditional particle reconstruction occurs at each location in the three-dimensional volume. Table 2 details the results of NTR by reporting the numbers of triplets reconstructed with and without NTR. On average, NTR recovers approximately 5% more triplets that had not been identified during the initial triplet search due to overlapping particle images.

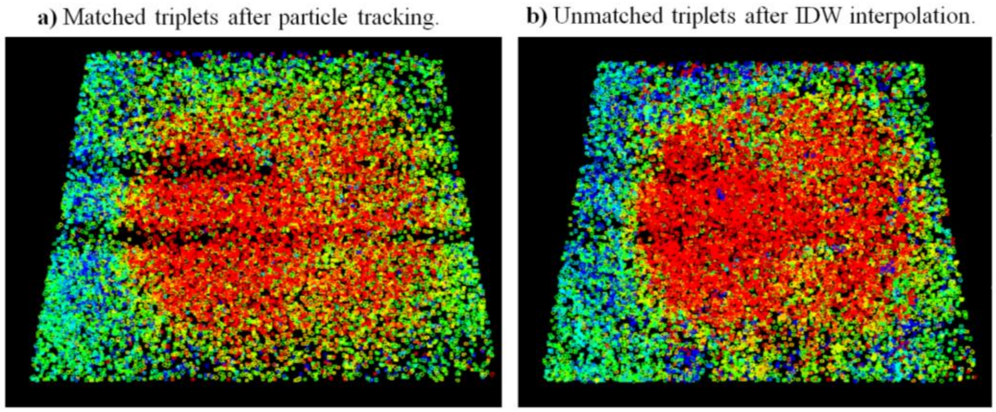

Figure 6. Triplets in three-dimensional space, colored by local velocity magnitude. a) Matched triplets. b) Unmatched triplets.

Table 1. Percentage increase in reconstructed triplets from six experimental cases using NTR.

Case ID	# Reconstructed Triplets Without NTR	# Reconstructed Triplets With NTR	% Overlapped particles
1	27,552	28,935	4.8%
2	16,504	16,639	0.8%
3	46,514	47,905	2.9%
4	124,553	136,630	8.8%
5	13,414	14,053	4.5%
6	84,003	90,571	7.3%
Average			**4.9%**

3.3 Auto-Calibration Reconstruction (ACR)

Lastly, we used experimental data from a fully developed turbulent channel flow to perform a preliminary performance test of the ACR method's ability to reduce calibration error. Figure 6 displays the local y-component of calibration error (x-component was similar, but is not shown) at arbitrarily chosen sample points within the volume that were uniformly spaced at each calibration plane. Each row (i.e., a, b, c) in Figure 6 refers to a particular aperture, while each column refers to the number of ACR passes that were performed. Without ACR, the mean y-component of error is approximately 0.5 pixels for apertures #0 and #1, but only 0.1 pixels for aperture #2. The calibration error decreases during each ACR pass, ultimately reducing to $\leq$ 0.1 pixels for each aperture.

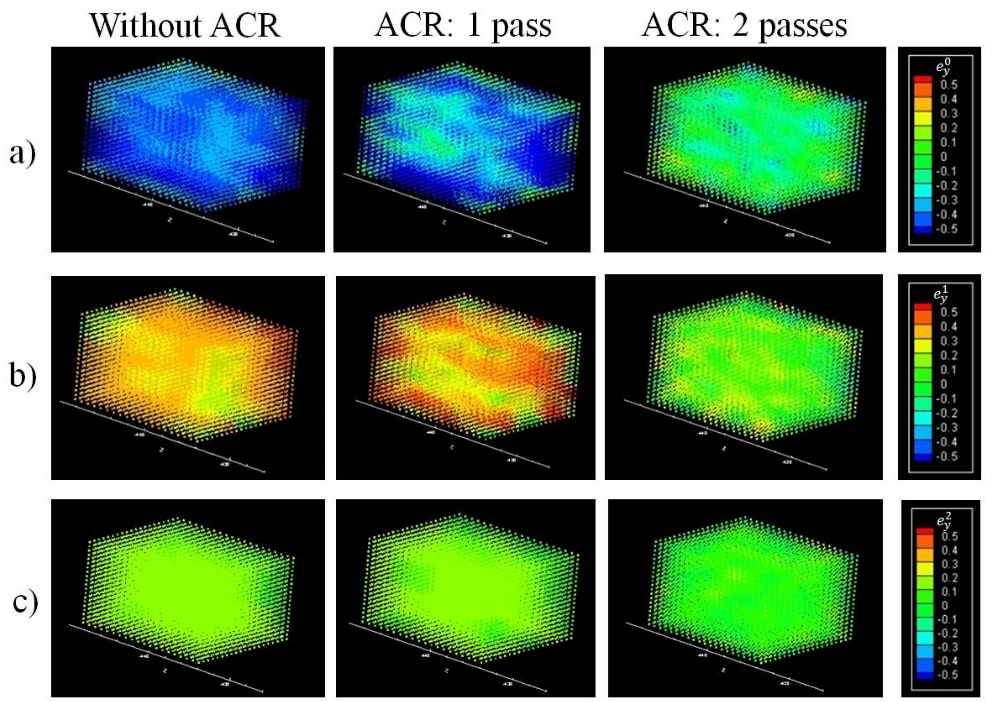

Figure 7. a) Aperture #0; b) Aperture #1; c) Aperture #2. Arbitrary sample points uniformly distributed at each calibration plane. Points are colored by the local y-component of error (pixels).

4. Summary

A study has been undertaken to increase triplet reconstruction yields by addressing three primary sources of common failure: non-uniform lighting, overlapping particle images, and calibration error. In regards to non-uniform lighting, Weak Particle Reconstruction (WPR) groups particle images in to weak and strong categories by prescribing a peak intensity threshold. During particle reconstruction, a triplet is created when two or more strong particles are matched at some three-dimensional position. Without WPR, a triplet requires all particle images to be strong type. As we have demonstrated, the use of weak particles can effectively reduce the number of lost triplets. To reconstruct triplets that have overlapping particle images, Neighbor Tracking Reconstruction (NTR) first identifies triplets that were not matched during particle tracking. For each, a search location was calculated using velocities from neighboring matched triplets, which was computed using an IDW point cloud interpolation function. We examined six existing experimental data sets and determined that NTR was able to reconstruct 5% more triplets, on average. Finally, Auto-Calibration Reconstruction (ACR) was used to address lost particles do to calibration error. The method was shown to effectively reduce calibration error in experimental data.

5. References

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